

The government budget constraint and debt dynamics

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Abstract

How the debt ratio moves with the primary balance, the interest rate and growth; the r minus g arithmetic that organises every fiscal debate.

Every question in this course, from multipliers to the euro crisis, comes back to one identity. The government finances what it does not raise in taxes by borrowing, and what it borrowed yesterday costs interest today. Written carefully, that identity tells you what “sustainable” can mean, why a country can run deficits forever, and why the same deficit is harmless at one interest rate and dangerous at another.

The flow identity

Let B_t be nominal debt at the end of year t , i_t the average nominal interest rate paid on it, G_t spending excluding interest, and T_t revenue. Then

$$B_t = (1 + i_t) B_{t-1} + G_t - T_t.$$

The difference $G_t - T_t$ is the **primary deficit**: what the state spends on everything but debt service, minus what it collects. Interest is kept apart because it is inherited, not chosen this year.

Ratios to GDP

Nobody reasons about debt in francs or euros; what matters is debt relative to the tax base, and the natural proxy for the tax base is nominal GDP Y_t . Divide equation () by Y_t and write $b_t = B_t/Y_t$, $d_t = (G_t - T_t)/Y_t$, and nominal growth $\gamma_t = Y_t/Y_{t-1} - 1$:

$$b_t = \frac{1 + i_t}{1 + \gamma_t} b_{t-1} + d_t \approx b_{t-1} + (r_t - g_t) b_{t-1} + d_t,$$

where r is the real interest rate and g real growth (inflation cancels to first order). Equation () is the workhorse. The change in the debt ratio has two parts: the primary deficit, and the **snowball effect** $(r - g) b_{t-1}$, the interest bill net of the erosion that growth provides.

Definition 1 (Debt-stabilising primary balance). The primary surplus that keeps the debt ratio constant: setting $b_t = b_{t-1} = b$ in () gives $-d = (r - g) b$, so the required primary surplus is $s^* = (r - g) b$. With $r > g$ a positive surplus is needed; with $r < g$ the ratio falls even with a small primary deficit.

Two numbers make the point. A country with debt at 100 percent of GDP and $r - g = 2$ percentage points must run a primary surplus of 2 percent of GDP every year just to stand still. The same country with $r - g = -1$ can run a primary deficit of 1 percent of GDP and watch the ratio drift down.

The intertemporal constraint

Iterate () forward and, provided $r > g$, the debt ratio today equals the present value of future primary surpluses:

$$b_{t-1} = \sum_{k=0}^{\infty} \left(\frac{1+g}{1+r} \right)^{k+1} s_{t+k} + \lim_{K \rightarrow \infty} \left(\frac{1+g}{1+r} \right)^K b_{t+K}.$$

The last term is the **transversality condition**: if it is zero, debt is eventually paid down by surpluses; if it is not, the government is rolling over debt and interest forever, a Ponzi scheme. When $r < g$ the discount factor exceeds one and the sum need not converge, which is exactly why the sign of $r - g$ dominates the sustainability debate in Is public debt sustainable?.

What the identity does not say

The identity is an accounting statement, not a theory. It says nothing about which r the market will charge, which depends on how much debt investors are willing to hold at a given yield, and it says nothing about g , which fiscal policy itself affects. Hall and Sargent decomposed the post-war fall in the US debt ratio and found that growth and the real rate, not primary surpluses, did most of the work (Hall & Sargent, 2011). That is the empirical face of the snowball term. The lessons that follow fill in the behaviour: how surpluses respond to debt, what spending does to output, and what happens when the central bank and the treasury do not agree about who adjusts.

Hall, G. J., & Sargent, T. J. (2011). Interest rate risk and other determinants of post-WWII US government debt/GDP dynamics. *American Economic Journal: Macroeconomics*, 3(3), 192–214. <https://doi.org/10.1257/mac.3.3.192>